

Effects of new physics to the the ρ -parameter in the supersymmetric standard model

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Abstract

The contribution to the ρ -parameter of the quark-Higgs sector of the supersymmetric standard model is computed non-perturbatively using the large- N_F expansion for the case $\tan\beta=0$. An explicit formula is found for the ρ -parameter which is ill-defined unless the triviality cutoff is taken into account. The cutoff dependence of the ρ -parameter is found to be large if and only if the top mass is larger than $2v$ so that the cutoff scale is of the order of the electroweak scale. These non-universal effects are the reflections at low energies of the physics beyond the supersymmetric standard model. By also considering the effects of the soft-breaking terms, cutoff effects in both decoupling and non-decoupling effects are analyzed.

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1. Introduction

The supersymmetric standard model is a natural generalization of the standard electroweak model that is perhaps realized in nature[1]. However, to this date, no supersymmetric partners of ordinary particles have been observed and they are assumed to be heavy enough to have avoided detection so far. When the mass of a particle is generated by a coupling constant, as is the case in the (supersymmetric) standard model, the decoupling theorem of [2] does not apply and the physical effects of this particle need not decrease as the particle becomes heavier. Using this fact, some important perturbative restrictions have been placed on yet unseen particles in the (supersymmetric) standard model, such as the top [3][4][5]. As the coupling constant becomes stronger, the perturbation theory becomes less reliable so that one also needs to analyze the behavior of non-decoupling effects outside the perturbative regime.

This problem raises a number of conceptually interesting questions: The ρ parameter increases with the fermion mass; what happens when we keep on increasing the fermion mass? Does it keep on increasing or somehow saturate? Is the perturbative bound on the top mass legitimate when we also consider the non-perturbative regime? In which region is the perturbation theory valid? What are the non-perturbative effects to the ρ parameter, or aren't there any? In the standard model, these questions were answered using the $1/N_F$ expansion in [6]. (For the Higgs case, also see [7], in regards to the S parameter, see [8].) First, we need to remember that the theory has a physical cutoff, namely the triviality scale, and that the fermion mass can only be at most a few times the symmetry breaking scale [9][10][11]. It was found that when the fermion mass increases to few times the vacuum expectation value ($\gtrsim 2.5v$, $v = 246$ GeV), the ρ -parameter depends substantially on how the cutoff is applied in the theory. This dependence is the sensitivity of the non-decoupling effects, such as the ρ parameter, to the physics beyond the standard model. In a renormalizable model, such as the standard model, it is at first sight surprising that a low energy parameter can depend on the details at high energies and this fact is special to non-decoupling effects.

In this work, we compute the contribution to the ρ -parameter from the supersymmetric Yukawa coupling non-perturbatively using the $1/N_F$ expansion. The effects of soft supersymmetric breaking terms will be included. In four dimensions, the only renormalizable supersymmetric interactions that may give rise to non-decoupling effects are the gauge and the Yukawa interactions. The gauge couplings in the case of the (supersymmetric) standard model have been measured to be small so that the Yukawa-Higgs coupling is perhaps of more interest. We will use the supersymmetric Yukawa-Higgs sector of the standard model solved in the large- N_F limit [12] to compute the ρ -parameter. We believe that our explicit results using the $1/N$ expansion clearly delineates the physics underlying the results. In the supersymmetric case, the formulation of supersymmetric theories on the lattice is still an open problem [13] so that it might be difficult to obtain non-perturbative information through other approaches.

In section 2, we explain precisely how our calculation applies to the supersymmetric standard electroweak model. We also explain how the large- N_F limit is taken and how the model is solved in this limit. In section 4, we compute the ρ -parameter in the case with no soft supersymmetry breaking terms and when the model is supersymmetric. This case is much simpler than the case including the effects of the soft breaking terms which is dealt with in section 5. We conclude with a discussion of our results in section 6.

2. The supersymmetric Yukawa–Higgs sector of the supersymmetric standard model in the large- N_F limit

In this section, we briefly explain how the quark–Higgs (or the lepton–Higgs) sector of the supersymmetric standard model is solved in the large- N_F limit. We refer to [12] for details.

Consider three scalar supermultiplets, Q_L, T_R and Φ which transform under $SU(N_F)_L \times U(1)_L \times U(1)_R$ as

$$Q_L \mapsto e^{i\alpha_L} U_L Q_L \quad \Phi \mapsto e^{i(\alpha_L - \alpha_R)} U_L \Phi \quad T_R \mapsto e^{i\alpha_R} T_R \quad (2.1)$$

where $U_L \in SU(N_F)$ and α_L, α_R are real. The boson, fermion and the auxiliary field components of the superfields Q_L, T_R and Φ are denoted $(\tilde{q}_L, q_L, F_{q_L}), (\tilde{t}_R, t_R, F_{t_R})$ and $(\phi, \tilde{\phi}, F_\phi)$ respectively.

The action of the model is derived from the standard kinetic term, the superpotential

$$W = y Q_L^\dagger \Phi T_R \quad (2.2)$$

and the soft supersymmetry breaking terms in the Lagrangian (in components)

$$-\mathcal{L}_{sb} = m_{\tilde{q}}^2 |\tilde{q}_L|^2 + m_{\tilde{t}}^2 |\tilde{t}_R|^2 \quad (2.3)$$

The model has global symmetry $SU(N_F)_L \times U(1)_R \times U(1)_L \times U(1)_R$ where the $U(1)_R$ R-symmetry acts on the fields as

$$\tilde{q}_L \mapsto e^{i\varphi_R} \tilde{q}_L \quad \tilde{t}_R \mapsto e^{i\varphi_R} \tilde{t}_R \quad \tilde{\phi} \mapsto e^{-i\varphi_R} \tilde{\phi} \quad F_{q_L} \mapsto e^{i\varphi_R} F_{q_L} \quad F_{t_R} \mapsto e^{i\varphi_R} F_{t_R} \quad (2.4)$$

while keeping the other fields fixed. The action of the rest of the transformations was defined in (2.1). The soft supersymmetry breaking terms in (2.3) are the most general ones while retaining this symmetry. Another soft breaking term, $\tilde{q}_L \phi \tilde{t}_R$ may be added which would break the $U(1)_R$ R-symmetry to \mathbb{Z}_2 symmetry. We shall not consider this term below.

In terms of components, the Lagrangian including the auxiliary fields is

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{kin} + \mathcal{L}_{pot} + \mathcal{L}_{sb} \\ -\mathcal{L}_{kin} &= |\partial\phi|^2 + \sum_{i=1}^{N_F} |\partial\tilde{q}_L^i|^2 + |\partial\tilde{t}_R|^2 + \frac{1}{2} \bar{\phi}\not{\partial}\phi + \bar{q}_L \not{\partial} q_L + \bar{t}_R \not{\partial} t_R \\ &\quad - \sum_{i=1}^{N_F} |F_{\phi_i}|^2 - \sum_{i=1}^{N_F} |F_{q_L^i}|^2 - |F_{t_R}|^2 \\ -\mathcal{L}_{pot} &= y \left[\bar{q}_L \phi t_R + \tilde{q}_L^\dagger \bar{\phi} \tilde{t}_R + \bar{q}_L \tilde{\phi} \tilde{t}_R \right] + y \left[\tilde{q}_L^\dagger \phi F_{t_R} + \tilde{q}_L^\dagger F_\phi \tilde{t}_R + F_{q_L}^\dagger \phi \tilde{t}_R \right] + \text{h.c.} \end{aligned} \quad (2.5)$$

where h.c. denotes the Hermitean conjugate terms.

The auxiliary fields may be integrated out to obtain a more familiar form of the Lagrangian

$$\begin{aligned}
-\mathcal{L} = & -\mathcal{L}_{sb} + |\partial\phi|^2 + \sum_{i=1}^{N_F} |\partial\tilde{q}_L^i|^2 + |\not{\partial}\tilde{t}_R|^2 + \frac{1}{2}\bar{\phi}\not{\partial}\phi + \bar{q}_L\not{\partial}q_L + \bar{t}_R\not{\partial}t_R \\
& + y \left[\bar{q}_L\not{\partial}t_R + \tilde{q}_L^\dagger\bar{\phi}t_R + \bar{q}_L\not{\partial}\tilde{t}_R + h.c. \right] + y^2 \left[|\phi|^2|\tilde{t}_R|^2 + \sum_{i=1}^{N_F} |\tilde{q}_L^i|^2|\tilde{t}_R|^2 + |\tilde{q}_L^\dagger\phi|^2 \right]
\end{aligned} \tag{2.6}$$

This model is a part of any supersymmetric generalization of the standard electroweak model in the quark–Higgs or the lepton–Higgs sector (for $N_F = 2$). Only one Yukawa coupling has been retained and the electroweak gauge couplings have been set to zero. To obtain the ρ -parameter to leading order in the gauge coupling constants, g, g' , it is only necessary to use external gauge fields. In this model, only one Higgs supermultiplet exists in the model so that there is no mixing of the Higgs multiplets. This case is often referred to as the $\tan\beta = 0$ case.

The vacuum expectation values of the superfields are

$$\langle\Phi_i\rangle = v/\sqrt{2}\delta_{i1} \quad \langle Q_L\rangle = \langle T_R\rangle = 0 \tag{2.7}$$

The vacuum expectation value spontaneously breaks the symmetry of the model to $SU(N_F - 1) \times U(1)_{L+R} \times U(1)_{\mathcal{R}}$.

The large- N_F limit is taken in this model by fixing v^2/N_F , y^2N_F while taking N_F to infinity. When no soft breaking terms are present, the leading order quantum effects that are of the same order as the classical terms may be summarized in the supergraphs in fig. 1

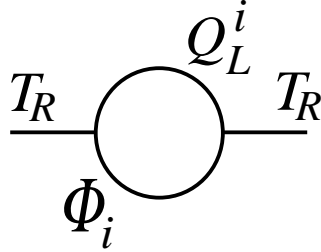


fig. 1 *Leading order contributions to the model when no soft supersymmetry breaking terms are present.*

The model is the natural supersymmetric generalization of the $O(N)$ model [14]. Including the effects of the soft breaking terms, the leading order effects may be neatly summarized in the following effective Lagrangian in which only the kinetic terms of the component fields in the T_R multiplet receive radiative corrections:

$$\begin{aligned}
\mathcal{L}^{eff} &= \mathcal{L}_{kin}^{eff} + \mathcal{L}_{pot} + \mathcal{L}_{sb} \\
-\mathcal{L}_{kin}^{eff} &= A_{1,bare} |\partial\tilde{t}_R|^2 + A_{2,bare} \bar{t}_R\not{\partial}t_R - A_{3,bare} |F_{t_R}|^2 + \text{other terms.}
\end{aligned} \tag{2.8}$$

where

$$\begin{aligned}
A_{1,bare}(p^2) &= 1 - \frac{y^2 N_F}{(4\pi)^2} \ln \frac{p^2}{s_{bare}} \\
A_{2,bare}(p^2) &= 1 - \frac{y^2 N_F}{2(4\pi)^2} \left[\ln \frac{p^2}{s_{bare}} + \ln \frac{m_{\tilde{q}}^2}{s_{bare}} + \left(1 + \frac{m_{\tilde{q}}^2}{p^2}\right)^2 \ln \left(1 + \frac{p^2}{m_{\tilde{q}}^2}\right) - \frac{m_{\tilde{q}}^2}{p^2} \right] \\
A_{3,bare}(p^2) &= 1 - \frac{y^2 N_F}{(4\pi)^2} \left[\ln \frac{m_{\tilde{q}}^2}{s_{bare}} + \left(1 + \frac{m_{\tilde{q}}^2}{p^2}\right) \ln \left(1 + \frac{p^2}{m_{\tilde{q}}^2}\right) \right]
\end{aligned} \tag{2.9}$$

in momentum space. s_{bare} denotes a regulator dependent number of dimension of mass squared whose explicit expression will not be necessary. (In dimensional regularization, $\ln s_{bare} = 1/\epsilon - \gamma_E + 2 + \ln(4\pi\mu^2)$, where γ_E is the Euler–Mascheroni constant and μ is the scale parameter introduced by dimensional regularization.) This characterization of the leading large- N_F results will prove to be useful in the subsequent computations. When there are no soft breaking terms $A_{i,bare}$ ’s are all equal since the fields $\tilde{t}_R, t_R, F_{t_R}$ all belong to the same supermultiplet.

We renormalize the coupling constant y at an arbitrary momentum squared scale s_0 as

$$y^2(s_0)N_F = \frac{y^2 N_F}{1 - y^2 N_F / (4\pi)^2 \ln s_0 / s_{bare}} \tag{2.10}$$

$m_{\tilde{q}}^2$ and v^2 need no renormalization to this order. Strictly speaking, $m_{\tilde{t}}^2$ is also renormalized, but this will not play a role in the following.

The full propagators that enter the calculations of physical quantities cease to make sense at some energy scales, the smallest of which we call the “triviality scale”. The triviality scale s_{triv} is determined through the equation $A_{3,bare}(s_{triv}) = 0$ and all particle masses need to be below this scale for consistency. s_{triv} is a physical scale independent of the renormalization scheme. Also, s_{triv} has the typical non-perturbative dependence on the coupling; for instance, when $m_{\tilde{q}}^2 = m_{\tilde{t}}^2 = 0$, $s_{triv} = s_0 \exp((4\pi)^2 / (y^2(s_0)N_F))$. The spectrum, which was arbitrary within perturbation theory, is restricted by the consistency of the theory. In particular, the mass of the fermion t can be at most a few times the scale v . For the possible spectra depending on the soft-breaking parameters, see [12].

3. The ρ -parameter

At energies well below the W, Z gauge boson masses, the interactions in the electroweak model may be effectively described by the current current interaction of the following form

$$-\mathcal{L}_{JJ} = \frac{G_F}{\sqrt{2}} \left(J_{\mu}^{+} J_{\mu}^{-} + \frac{1}{2} \rho J_{\mu}^0 J_{\mu}^0 \right) \tag{3.1}$$

Here, J_{μ}^{\pm}, J_{μ}^0 denote the charged and the neutral weak currents. ρ represents the relative strength of the neutral current interaction to the charged one. The weak interactions are mediated by W, Z gauge bosons whose massive modes are the Nambu–Goldstone bosons

of the ungauged theory. Consequently, the ρ -parameter may be characterized using the effective Lagrangian for the Nambu–Goldstone bosons χ^+, χ^0 as [15]

$$\rho = \frac{Z_{\chi^+}(p^2)}{Z_{\chi^0}(p^2)} \Big|_{p^2=0} \quad (3.2)$$

using the effective Lagrangian

$$-\mathcal{L}_{eff}^{NG} = Z_{\chi^+} \left| \partial_\mu \chi^+ - \frac{gv}{2} W_\mu^+ \right|^2 + \frac{1}{2} Z_{\chi^0} \left(\partial_\mu \chi^0 - \frac{gv}{2 \cos \theta_W} Z_\mu \right)^2 + \text{other terms} \quad (3.3)$$

This formula characterizes the deviation of the ρ -parameter from unity as the lack of custodial symmetry amongst the Nambu–Goldstone bosons. In the $1/N_F$ expansion, χ^+ may be taken to be any Nambu–Goldstone boson in the $N_F - 1$ multiplet of $SU(N_F - 1)$ due to the residual symmetry. These expressions (3.1), (3.2) are exact up to higher order terms in the electroweak gauge coupling constants and in the $1/N_F$ expansion.

4. ρ -parameter in the case with no soft breaking terms

In this section, we compute the ρ -parameter non-perturbatively in the case when $m_{\tilde{q}}^2 = m_{\tilde{t}}^2 = 0$. This case is simpler than the more general case dealt with in the next section yet illustrates some of the main concepts. The leading order corrections to the gauge boson propagators come from the graphs of $\mathcal{O}(1/N_F)$ in fig. 2 in the component formulation.

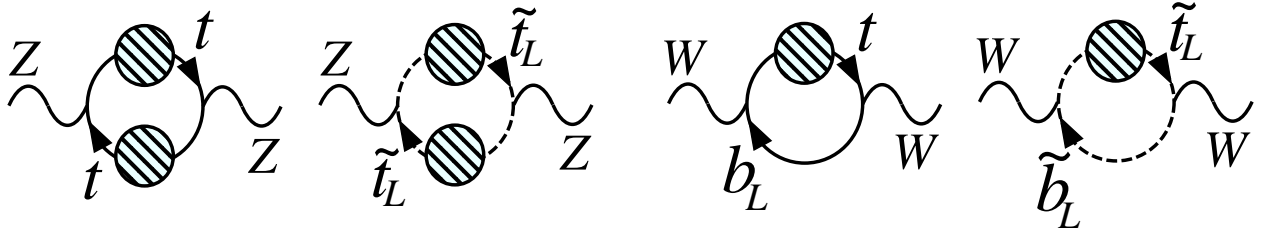


fig. 2 *Leading order quantum corrections to the Z, W propagators.*

In the figure, we collectively denoted $q_L^i \equiv b_L, \tilde{q}_L^i \equiv \tilde{b}_L$ ($i \neq 1$). There are also seagull contributions of the same order, however these contributions are the same for both W and Z so that they cancel in their contribution to the ρ parameter and we shall not discuss them here.

Using the full propagators in the large N_F limit derived from (2.8) and using the definition of the ρ parameter in (3.1), we arrive at the expression for the ρ parameter to leading order in the $1/N_F$ expansion:

$$\delta\rho = \frac{2\hat{m}^4}{v^2} \int_{k^2 < \Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 (A_1(k^2)k^2 + \hat{m}^2)^2} \quad (4.1)$$

We have used the renormalized parameters in this expression; here, $\hat{m} \equiv y(s_0)v/\sqrt{2}$ and A_1 is the same formula as $A_{1,bare}$ with the replacement, $y^2 \mapsto y^2(s_0)$, $s_{bare} \mapsto s_0$; i.e.,

$A_1(p^2) \equiv 1 - y^2(s_0)N_F/(4\pi)^2 \ln p^2/s_0$. Since this is the contribution from one multiplet, we need to multiply by the number of colors. The renormalization scale s_0 is arbitrary. Since we will only be interested in the behavior of the ρ parameter with respect to the particle masses which are both physical quantities, a change in the renormalization scale does not affect the results at all.

The integrand in this expression for the ρ -parameter is integrable both at $k^2 = 0$ and at $k^2 = \infty$. However, we cannot integrate over k^2 from zero to infinity due to the existence of a pole in the integrand. The location of this pole is always above the triviality scale s_{triv} so that we cutoff this integral at the scale $\Lambda^2 < s_{triv}$. This is also natural since the triviality scale is the intrinsic cutoff of the theory, above which scale the theory breaks down. The ρ -parameter now depends on how the cutoff is implemented and to understand the dependence of this parameter on the cutoff procedure, we varied the cutoff scale as $\Lambda/\sqrt{s_{triv}} = 0.1, 0.5, 0.9$. The result is shown in fig. 3 as the variation of the ρ -parameter against the physical mass of the particles in the T_L, T_R supermultiplets which are degenerate in mass. The mass of the particle is determined from the poles in the propagators derived from the full Lagrangian (2.8). We also plot the one and two loop terms in the coupling constant expansion of (4.1). The cutoff dependence in the ρ parameter can be seen from (4.1) to be of order v^2/Λ^2 .

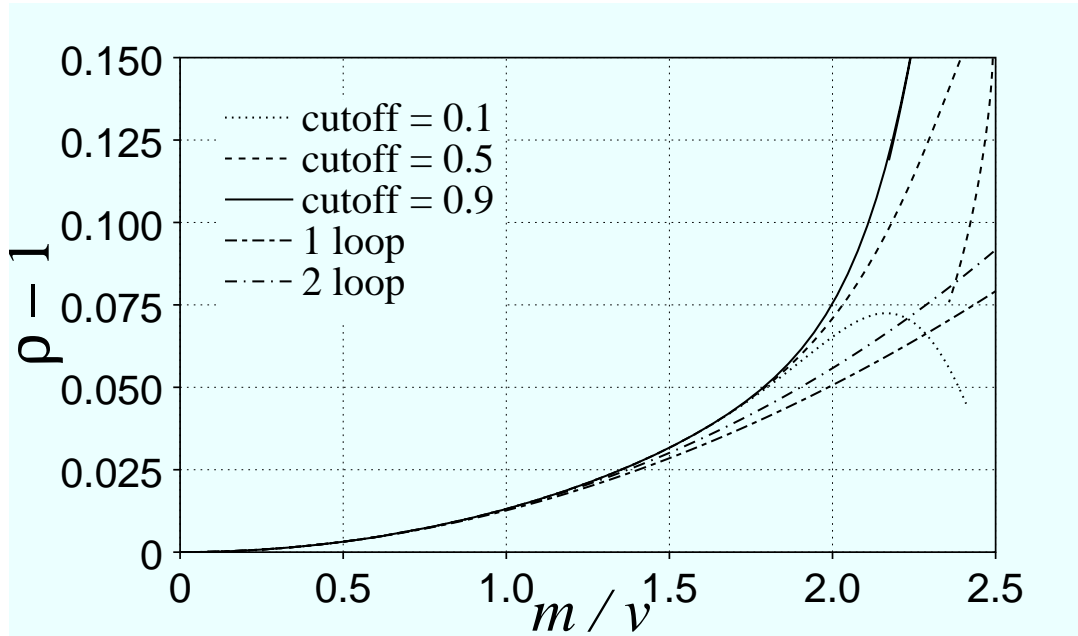


fig. 3 $\delta\rho$ vs. mass of $t, \tilde{t}_L, \tilde{t}_R$ in units of v .

Here and below, we will plot only the region where the cutoff is larger than the mass. Also, we will set $N_F = 2$ in the plots to put them in a more familiar context. For other values of N_F , just rescale what we call v^2 . The ρ parameter increases with the fermion mass. Beyond a certain mass ($\sim 2v$), the relative size of the cutoff effects become substantial. Non-perturbatively, there is a restriction on the mass of the fermion so that it is smaller than $2.49v$.

If we expand the non-perturbative expression for the ρ -parameter in (4.1) in powers of the coupling constant, each term in the expansion is finite without any need for a cutoff. Lowest order terms in this expansion up to three loop order are

$$\delta\rho = 2x + 4N_F x^2 + 8N_F^2 \left(\frac{\pi^2}{3} + 1 \right) x^3 + \mathcal{O}(x^4) \quad \text{where} \quad x \equiv \frac{G_F m_{t,pert}^2}{8\sqrt{2}\pi^2} \quad (4.2)$$

Here we defined the perturbative mass $m_{t,pert}^2 \equiv y^2(m_{t,pert}^2)v^2/2$.

An interesting question we may ask, given the above results, is how good or how bad perturbation theory performs in our model. As $\delta\rho$ increases, the perturbative result becomes less reliable. However, the cutoff effects that inevitably arise in the non-perturbative result increases. As we increase the mass, at some point the non-universal effects become as large as the deviation of the one loop result from the non-perturbative result. Beyond this point, the one loop result is as reliable (or is unreliable) as the non-perturbative result. Up to this point, the one loop result deviates from the non-perturbative result by at most 50%. It is also interesting to analyze the significance of the higher loop contributions: As the mass increases, the higher loop effects become more important and they improve the agreement of the perturbative results with the non-perturbative one. At some point, however, the non-universal effects become as large as the improvement the higher loop contribution makes. We find that the two loop term contribution is at most 7% of $\delta\rho$ before the cutoff effects become as large. For the three loop contribution, this number is 2%. These numbers are products of a rather crude approximation and cannot be taken too literally; however, we believe that they illustrate the role of the perturbative approximation to non-decoupling effects in a theory with a triviality scale. The appearance of inherent ambiguities in the non-perturbative result, of course, is a limitation of the (supersymmetric) standard model. Once we have a theory that is well defined in the ultraviolet that reduces to the standard model in the infrared, the non-universal effects will disappear.

The ρ parameter may also be computed from the radiative corrections to the two point functions of the Nambu-Goldstone bosons as in (3.2). The result is exactly the same as the one obtained via the gauge bosons in (4.1). The one particle irreducible contributions to the two point functions of Nambu-Goldstone bosons χ^0, χ^+ are listed in fig. 4

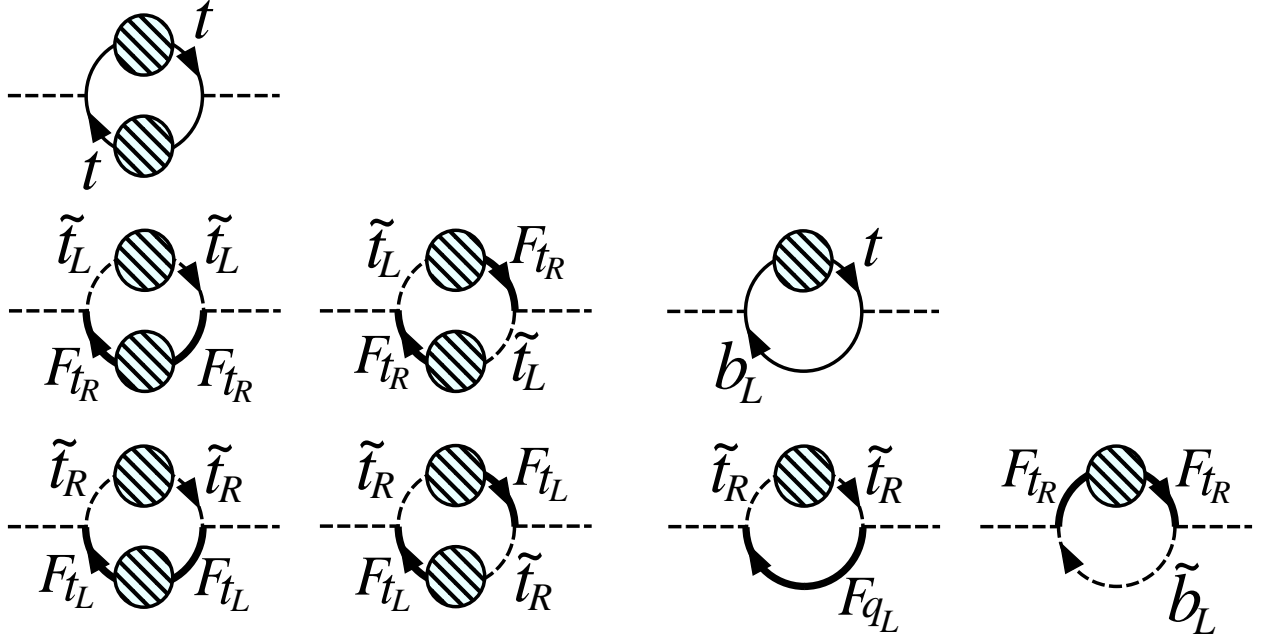


fig. 4 The leading order quantum corrections to the two point functions of the Nambu-Goldstone bosons, χ^0, χ^+ . The left five graphs correct χ^0 , the right three correct χ^+ .

We have used the notations $q_L^1 \equiv t_L, \tilde{q}_L^1 \equiv \tilde{t}_L, F_{q_L}^1 \equiv F_{t_L}$. We obtain more familiar looking graphs if we integrate out the auxiliary fields. However, it seems difficult to directly sum the contributions corresponding to the graphs in fig. 4 once we integrate out the auxiliary fields.

Finally, we note that the non-perturbative expression for the ρ parameter (4.1) is identical to that of the standard model [6], with the replacement $y^2(s_0) \mapsto 2y^2(s_0)$. Naively, this looks like twice the standard model case, however we caution that this is not the case, since the full propagators themselves that enter the computation contain contributions from the full supermultiplet.

5. ρ -parameter including the effects of the soft supersymmetry breaking terms

In this section, we compute the ρ -parameter in the supersymmetric quark-Higgs sector including the effects of the soft supersymmetry breaking terms. The ρ parameter may be obtained from the graphs in fig. 2 in a calculation similar to that of the last section, albeit more complicated:

$$\delta\rho = \frac{\hat{m}^4}{v^2} \int_{k^2 < \Lambda^2} \frac{d^4k}{(2\pi)^4} \left[\frac{1}{k^2(A_2(k^2)k^2 + \hat{m}^2)^2} + \frac{k^2}{(k^2 + m_q^2)^2 \left(A_3(k^2)(k^2 + m_q^2) + \hat{m}^2 \right)^2} \right] \quad (5.1)$$

As in the previous section, the renormalized parameters are used in this expression and A_i 's are the same formulas as $A_{i,bare}$'s in (2.9) with the replacement, $y^2 \mapsto y^2(s_0)$, $s_{bare} \mapsto s_0$. The renormalization scale s_0 is arbitrary and does not affect the results below.

As in the case with no soft breaking terms, this expression for the ρ -parameter requires a cutoff Λ^2 to give a finite answer. We have plotted the ρ parameter against the physical mass of the fermion, t , for the cutoff values $\Lambda/\sqrt{s_{triv}} = 0.1, 0.5$ and 0.9 to see the cutoff dependence of the expression (5.1) in fig. 5.

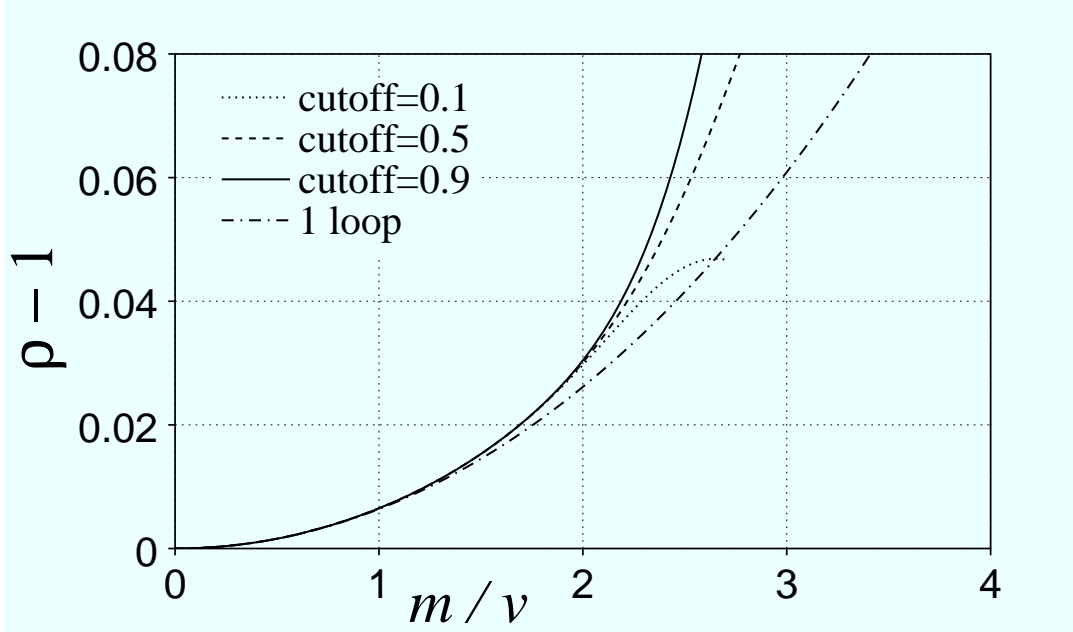


fig. 5 The parameter $\delta\rho$ against the mass of t in units of v for the value $m_q^2 = (4\pi v)^2/N_F$.

The soft-breaking scale has been set to $m_q^2 = (4\pi v)^2/N_F$ so that its contribution is of the same order as the contribution from the symmetry breaking scale. If we expand the expression in powers of coupling for $\delta\rho$ in (5.1), there is no need to impose a cutoff. The one loop term in this expansion is

$$\delta\rho = 2x \left[1 + y + y(1+y) \ln \frac{y}{1+y} \right] + \mathcal{O}(x^2) \quad (5.2)$$

Here x was defined in (4.2) and $y \equiv m_q^2/\hat{m}^2$. The one loop term, of course, agrees with the previous literature [4] and is shown in the plot fig. 5.

The qualitative behavior of the ρ parameter is the same as the previous section; as the fermion mass increases, the ρ parameter also increases. Beyond a certain mass ($\sim 2v$) the non-universal effects become appreciable. The maximum fermion mass in this case is $3.58v$. The non-universal effects in the ρ parameter may be obtained from (5.1) to be of $\mathcal{O}(v^2/\Lambda^2)$. It is important to note that the leading order cutoff dependence on m_q^2 is further suppressed and is of $\mathcal{O}(v^2 m_q^2 \ln(m_q^2)/\Lambda^4)$. Therefore the non-universal effects are appreciable if and only if the fermion mass is large (or the cutoff is not so high). In essence, increasing the coupling constant gives rise to non-decoupling effects and increasing the soft breaking scale gives rise to decoupling effects.

As in the case without the supersymmetry breaking terms, we may compute the ρ parameter to leading order using the propagators for the Nambu-Goldstone bosons by

incorporating the corrections from the graphs in fig. 4:

$$\begin{aligned} \delta\rho = & \frac{2\hat{m}^4}{v^2} \int_{k^2 < \Lambda^2} \frac{d^4k}{(2\pi)^4} \left[\frac{(A_2(k^2)k^2)'}{(A_2(k^2)k^2 + \hat{m}^2)^3} \right. \\ & \left. + \frac{1}{(k^2 + m_{\tilde{q}}^2) \left(A_3(k^2)(k^2 + m_{\tilde{q}}^2) + \hat{m}^2 \right)^2} \left(1 - \frac{k^2(A_3(k^2)(k^2 + m_{\tilde{q}}^2))'}{A_3(k^2)(k^2 + m_{\tilde{q}}^2) + \hat{m}^2} \right) \right] \end{aligned} \quad (5.3)$$

The primes denote the derivative with respect to k^2 . This expression is the same as the expression (5.1) obtained using the current–current correlation function to all orders in perturbation theory. The two expressions (5.1), (5.3) differ by cutoff effects of order $v^2 m_{\tilde{q}}^2 / \Lambda^4$, which are non–perturbative. This is only to be expected; once the result is non–universal, physical results are well defined only up to cutoff effects. Consequently, in general two methods of computation for the same quantity will yield results which are exactly the same to all orders in perturbation theory, but not necessarily the same in the non–universal effects. (Hence, of course, the name “non–universal”.)

6. Discussion of the results

Our objective was to obtain a non–perturbative expression for a non–decoupling effect, the ρ –parameter, in the supersymmetric standard model and to analyze it. Considering that the non–decoupling effects become appreciable only when the coupling becomes large, this is both a natural and an important problem. A non–perturbative expression was obtained including the effects of the soft breaking terms using the large– N_F expansion. When we try to evaluate this expression, we find that the expression is not finite unless we take into account the fact that the theory, non–perturbatively, contains an intrinsic cutoff, namely the triviality scale. Taking this into account and evaluating the expression, we find that the result is cutoff–dependent when the fermion mass is large or, equivalently, when the triviality cutoff is small. That such a seemingly simple closed form expression is non–perturbative and contains so much physical information, we find rather remarkable.

Had the cutoff effects appeared for observables measured at energies close to the cutoff, it would not have been surprising; what is surprising is that the cutoff effects appear in a low energy physically measurable parameter. The naive expectation for the cutoff effects would have been of $\mathcal{O}(p^2/\Lambda^2)$ where p^2 is the momentum scale of interest — in this case zero. This naive expectation fails and in fact the cutoff dependence is of $\mathcal{O}(v^2/\Lambda^2)$. This is special to non–decoupling effects; non–decoupling effects, though they are measured at low energies, effectively probe the physics at the electroweak symmetry breaking energy scale, v^2 . Therefore when the fermion mass becomes heavy and the coupling large, the cutoff becomes of $\mathcal{O}(v^2)$ so that the cutoff dependence also becomes appreciable. In the real world, the cutoff scale corresponds to the energy scale of new physics that appears beyond the standard model. As the mass of the top becomes higher, the energy scale of new physics becomes lower and the ρ –parameter, along with other non–decoupling parameters, becomes more sensitive to the new physics that appears.

By contrast, decoupling effects do not see the physics of the symmetry breaking scale. The cutoff dependence of the ρ parameter is of order v^2/Λ^2 and no contributions of

$\mathcal{O}(m_{\tilde{q}}^2/\Lambda^2)$ exist. The cutoff dependence may arise only when the coupling is large and the cutoff scale is not so high compared to the symmetry breaking scale; it cannot arise when the soft breaking scale is of the order of the cutoff scale and when v^2 is far below it. Since the soft breaking terms considered here are not interaction terms, we expect that they do not to affect low energy physics when we increase it, even to the order of the cutoff. The results confirm this picture. However, since the cutoff effects are non-perturbative in essence, we know of no theorems that apply to this case.

We also analyzed the reliability of the perturbative approximation to $\delta\rho$ in comparison with the non-perturbative result. For instance, we found that the two loop result improves the one loop result by 10% at best, since when the two loop contribution is any larger, the non-universal effect is at least as large.

A small $\delta\rho$, as seen in nature, may be achieved by a fermion mass in the perturbative regime. There is perhaps another intriguing possibility that the scale of new physics is of order v^2 and the physics beyond the standard model manifests in the particular form of non-universal effect so as to make the $\delta\rho$ small. This would require very specific properties of higher energy physics and it is not clear that such a model may be constructed.

Clearly, there is further work to be done: It would be interesting to study more general models, such as ones with more complicated Higgs sectors. Also, it would be interesting to explicitly construct a model that corresponds to the physics beyond the triviality cutoff and see how the non-decoupling effects depend on the particular physics chosen at the higher energy scales. The issue of the non-perturbative behavior of non-decoupling effects is of interest on its own. To apply this to the standard model ($N_F = 2$), $1/N_F$ is *a priori* not a small parameter so that it is important to study the problem using other non-perturbative approaches as well. An interesting task would be to find a more systematic understanding of non-decoupling effects in general.

The physics of non-decoupling effects is qualitatively similar for the supersymmetric and the non-supersymmetric case. This is perhaps surprising considering how the quantum properties of supersymmetric theories markedly differ from that of non-supersymmetric theories. However, when we understand the underlying physics, it is consistent with the behavior of decoupling effects, non-decoupling effects and their cutoff dependence.

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